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# RECOGNITION OF THE GROUP $E_6(2)$ BY GRUENBERG–KEGEL GRAPH

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The Gruenberg–Kegel graph (or the prime graph) of a finite group $G$ is a simple graph $\Gamma(G)$ whose vertices are the prime divisors of the order of $G$, and two distinct vertices $p$ and $q$ are adjacent in $\Gamma(G)$ if and only if $G$ contains an element of order $pq$. A finite group is called recognizable by Gruenberg–Kegel graph if it is uniquely determined up to isomorphism in the class of finite groups by its Gruenberg–Kegel graph. In this paper, we prove that the finite simple exceptional group of Lie type $E_6(2)$ is recognizable by its Gruenberg–Kegel graph.

Keywords: finite group, simple group, exceptional group of Lie type, Gruenberg–Kegel graph (prime graph).

**Распознаваемость группы $E_6(2)$ по графу Грюнберга — Кегеля. В. Го, А. С. Кондратьев, Н. В. Маслова.**

Граф Грюнберга — Кегеля (или граф простых чисел) конечной группы $G$ — это обыкновенный граф $\Gamma(G)$, в котором вершинами служат простые делители порядка группы $G$ и две различные вершины $p$ и $q$ смежны тогда и только тогда, когда $G$ содержит элемент порядка $pq$. Конечная группа называется распознаваемой по графу Грюнберга — Кегеля, если она однозначно с точностью до изоморфизма определяется своим графом Грюнберга — Кегеля в классе конечных групп. В этой работе мы доказываем, что конечная простая исключительная группа лиева типа $E_6(2)$ распознаваема по графу Грюнберга — Кегеля.

Ключевые слова: конечная группа, простая группа, исключительная группа лиева типа, граф Грюнберга — Кегеля (граф простых чисел).

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Our terminology and notation are mostly standard and can be found, for example, in [1; 3; 5]. Throughout the paper we consider only finite groups and simple graphs, and henceforth the term group means finite group, the term graph means simple graph (undirected graph without loops and multiply edges).

Let $G$ be a group. Denote by $\omega(G)$ the *spectrum* of $G$, i. e., the set of all element orders of $G$. The *Gruenberg–Kegel graph* (or the *prime graph*) of $G$ is a simple graph $\Gamma(G)$ whose vertices are the prime divisors of the order of $G$, and two distinct vertices $p$ and $q$ are adjacent in $\Gamma(G)$ if and only if $pq \in \omega(G)$.

In the finite group theory, the direction of the study of the recognizability of finite groups by spectrum or Gruenberg–Kegel graph is still dynamically developed. A group $G$ is *recognizable by spectrum* (or *by Gruenberg–Kegel graph*), if it is uniquely determined up to isomorphism in the class of finite groups by its spectrum (or by Gruenberg–Kegel graph, respectively). It is clear that if a group is recongizable by Gruenberg–Kegel graph, then the group is recognizable by spectrum, but in general case the converse does not hold. For a group $G$ which is not recognizable by its Gruenberg–Kegel graph, the question on the structure of groups with the same Gruenberg–Kegel graphs as $\Gamma(G)$ is of interest. In this paper, we make a one more step in the solution of the problem of recognition of a simple groups by Gruenberg–Kegel graph. We prove the following theorem.

**Theorem.** *The finite simple exceptional group of Lie type $E_6(2)$ is recognizable by its Gruenberg–Kegel graph.*

If $n$ is a positive integer, then denote by $\pi(n)$ the set of all prime divisors of $n$.

Let $G$ be a group. The spectrum $\omega(G)$ is partially ordered with respect to divisibility and is uniquely determined by the set $\mu(G)$ of its maximal elements. Denote by $s(G)$ the number of connected components of the graph $\Gamma(G)$, by $\{\pi_i(G) \mid 1 \leq i \leq s(G)\}$ the set of connected components of $\Gamma(G)$ (if $|G|$ is even, then we assume that $2 \in \pi_1(G)$). Let $\pi(G) = \pi(|G|)$ and $\mu_i(G) = \{n \in \mu(G) \mid \pi(n) \subseteq \pi_i(G)\}$ for $1 \leq i \leq s(G)$. If $\pi$ is a set of primes, then $G$ is a $\pi$-group if $\pi(G) \subseteq \pi$.

A set of pairwise non-adjacent vertices of a graph is called its *coclique*. Denote by $t(G)$ the maximal size of a coclique of $\Gamma(G)$ and by $t(r, G)$, where $r$ is a prime, the maximal size of a coclique of $\Gamma(G)$ which contains the vertex $r$.

Let $V$ be a $G$-module over a field. Remind that a non-trivial element $g \in G$ *acts fixed-point-free* on $V$ if $g$ does not fix non-zero vectors from $V$.

To prove the theorem we need the following series of lemmas.

**Lemma 1** (the Gruenberg–Kegel theorem, see [12, Theorem A]). *Let $G$ be a group with disconnected Gruenberg–Kegel graph. Then one of the following statements holds*:

(a) *$G$ is a Frobenius group*;
(b) *$G$ is a 2-Frobenius group, i. e., $G = ABC$, where $A$ and $AB$ are normal subgroups of $G$, $AB$ and $BC$ are Frobenius groups with kernels $A$ and $B$ and complements $B$ and $C$, respectively*;
(c) *$G$ is an extension of a nilpotent $\pi_1(G)$-group by a group $A$, where $P \cong \mathrm{Inn}(P) \leq A \leq \mathrm{Aut}(P)$, $P$ is a non-abelian simple group with $s(G) \leq s(P)$, and $A/\mathrm{Inn}(P)$ is a $\pi_1(G)$-group.*

**Lemma 2** (see [4;6;7;10–12]). *Let $P$ be a non-abelian simple group with disconnected Gruenberg–Kegel graph such that $t(P) \geq 4$ and $t(2, P) \geq 4$. Then the following statements hold*:

(a) *$|\mu_i(P)| = 1$ for $i > 1$, and $n_i(P)$ denotes a unique element from $\mu_i(P)$, where $i > 1$*;
(b) *$P$, $t(P)$, $t(2, P)$, and the set $\{n_i(P) \mid 2 \leq i \leq s(P)\}$ can be found in Table* 1; *here $p$ is an odd prime.*

**Lemma 3** (see [12]). *Let $G$ be a group with disconnected Gruenberg–Kegel graph such that $G$ is not isomorphic to a Frobenius group and a 2-Frobenius group, and let $P$ is a unique non-abelian composition factor of $G$. Then, for each $i \in \{2, \ldots, s(G)\}$, there exists $j \in \{2, \ldots, s(P)\}$ such that $\pi_i(G) = \pi_j(P)$.*

**Lemma 4** (see [9, Theorem 1]). *Let $G$ be a group with $t(G) \geq 3$ and $t(2, G) \geq 2$. Then the following statements hold*:

(a) *$G$ is an extension of a solvable group $K$ by a group $A$ with non-abelian simple socle $P$*;
(b) *if $\rho$ is a coclique of the graph $\Gamma(G)$, then $|\rho \cap (\pi(K) \cup \pi(A/P))| \leq 1$, in particular, $t(P) \geq t(G) - 1$*;
(c) *one of the following two statements holds*:
  (i) *any prime $r \in \pi(G)$, which is non-adjacent to 2 in $\Gamma(G)$, does not divide $|K||A/P|$, in particular, $t(2, P) \geq t(2, G)$*;
  (ii) *there is a prime $r \in \pi(K)$, which is non-adjacent to 2 in $\Gamma(G)$, $P$ is isomorphic to $A_7$ or $A_1(q) \cong PSL_2(q)$ for some odd $q$, $t(G) = 3$, and $t(2, G) = 2$.*

**Lemma 5** (Gerono's lemma, see [2]). *Let $p$ and $q$ be primes such that $p^a - q^b = 1$ for some positive integers $a$ and $b$. Then the pair $(p^a,\ q^b)$ is one of the following*: $(3^2,\ 2^3)$, *$(p,\ 2^b)$, or $(2^a,\ q)$, where $a$ is a prime and $b$ is a power of* 2.

**Lemma 6** (Zsigmondy's theorem, see [14]). *Let $q \geq 2$ and $n \geq 3$ be integers. If $(q, n) \neq (2, 6)$, then there exists a prime $r$ such that $r$ divides $q^n - 1$ and does not divide $q^i - 1$ for each $i < n$.*

In notation system of Lemma 6, a prime $r$ which divides $q^n - 1$ and does not divide $q^i - 1$ for each $i < n$ is called a *primitive prime divisor* of the number $q^n - 1$.

**Table 1. Simple groups $P$ with $s(P) \geq 2$, $t(p) \geq 4$ and $t(2,P) \geq 4$**

| $P$ | Restrictions to $p$ and $q$ | $t(P)$ | $t(2,P)$ | $n_2(P), \ldots, n_{s(P)}(P)$ |
|---|---|---|---|---|
| $A_2(q)$ | $(q-1)_3 = 3,\ q = 2^m > 4$ | 4 | 4 | $\frac{q^2+q+1}{(3,q-1)}$ |
| ${}^2A_2(q)$ | $(q+1)_3 = 3,\ q = 2^m > 8$ | 4 | 4 | $\frac{q^2-q+1}{(3,q+1)}$ |
| ${}^2D_n(q)$ | $n = 2^m \geq 4$, $q$ is even, $(n,q) \neq (4,2)$ | $\left[\frac{3n+4}{4}\right]$ | 4 | $q^n+1$ |
| $E_6(q)$ | $q$ is even | 5 | 4 | $\frac{q^6+q^3+1}{(3,q-1)}$ |
| ${}^2E_6(q)$ | $q = 2^m \geq 4$ | 5 | 4 | $\frac{q^6-q^3+1}{(3,q+1)}$ |
| ${}^2F_4(q)$ | $q = 8$ | 4 | 4 | $37,\ 109$ |
| | $q = 2^{2m+1} > 8$ | 5 | 4 | $q^2-\sqrt{2q^3}+q-\sqrt{2q}+1$, $q^2+\sqrt{2q^3}+q+\sqrt{2q}+1$ |
| $E_7(2)$ | | 8 | 5 | $73,\ 127$ |
| $M_{23}$ | | 4 | 4 | $11,\ 23$ |
| $F_3$ | | 5 | 4 | $19,\ 31$ |
| $A_2(4)$ | | 4 | 4 | $3,\ 5,\ 7$ |
| ${}^2B_2(q)$ | $q = 2^{2m+1} > 2$ | 4 | 4 | $q-1,\ q-\sqrt{2q}+1$, $q+\sqrt{2q}+1$ |
| ${}^2E_6(2)$ | | 5 | 4 | $13,\ 17,\ 19$ |
| $E_8(q)$ | $q \equiv 2,3 \pmod 5$ | 12 | 5 | $\frac{q^{10}+q^5+1}{q^2+q+1}$, $q^8-q^4+1$, $\frac{q^{10}-q^5+1}{q^2-q+1}$ |
| $M_{22}$ | | 4 | 4 | $5,\ 7,\ 11$ |
| $J_1$ | | 4 | 4 | $7,\ 11,\ 19$ |
| $O'N$ | | 5 | 4 | $11,\ 19,\ 31$ |
| $LyS$ | | 6 | 4 | $31,\ 37,\ 67$ |
| $Fi'_{24}$ | | 6 | 4 | $17,\ 23,\ 29$ |
| $F_1$ | | 11 | 5 | $41,\ 59,\ 71$ |
| $E_8(q)$ | $q \equiv 0,1,4 \pmod 5$ | 12 | 5 | $\frac{q^{10}+q^5+1}{q^2+q+1}$, $\frac{q^{10}-q^5+1}{q^2-q+1}$, $q^8-q^4+1$, $\frac{q^{10}+1}{q^2+1}$ |
| $J_4$ | | 7 | 6 | $23,\ 29,\ 31,\ 37,\ 43$ |

**Lemma 7** (Mazurov's lemma, see [8, Lemma 1]). *Let $G$ be a group, $N$ be a normal subgroup of $G$ such that $G/N$ is a Frobenius group with kernel $F$ and cyclic complement $C$. If $(|F|,|N|) = 1$ and $F \not\leq NC_G(N)/N$, then $s|C| \in \omega(G)$ for some $s \in \pi(N)$.*

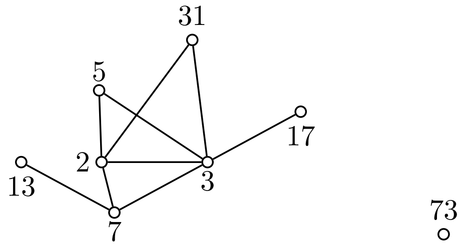
**Lemma 8** [13, Proposition 2]. *Let the group $G = {}^3D_4(q)$ act on a non-zero vector space $V$ over a field of characteristic coprime to $q$. Then each element of order $q^4-q^2+1$ from $G$ fixes a non-zero vector from $V$.*

The following easy-proving assertion is well-known.

**Lemma 9.** *Let $H$ be a simple group, $F$ be a field of characteristic $p > 0$, $V$ be an absolutely* *irreducible $FH$-module and $\beta$ be the Brauer character of $V$. If $g \in H$ with $(|g|, p) = 1$, then*

$$\dim C_V(g) = (\beta|_{\langle g\rangle}, 1|_{\langle g\rangle}) = \frac{1}{|g|}\sum_{x\in\langle g\rangle}\beta(x).$$

**Proof of the theorem.** Let $L = E_6(2)$. By [10; 11], the graph $\Gamma(L)$ is the following

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In particular, $s(L) = 2$, $|\pi(L)| = 8$, $\pi_1(L) = \{2, 3, 5, 7, 13, 17, 31\}$, $\pi_2(L) = \{73\}$, $t(L) = 5$, $t(2, L) = 4$, and the graph $\Gamma(L)$ has exactly two cocliques of maximal size, namely, $\{5, 7, 17, 31, 73\}$ and $\{5, 13, 17, 31, 73\}$.

Let $G$ be a group with $\Gamma(G) = \Gamma(L)$, and let $\overline{G} = G/F(G)$, where $F(G)$ is the Fitting subgroup of $G$. By Lemmas 1–4, we have $P \cong \mathrm{Inn}(P) \trianglelefteq \overline{G} \leq \mathrm{Aut}(P)$, where $P$ is a non-abelian simple group such that $s(P) \geq 2$, $\pi(F(G)) \cup \pi(\bar{G}/\mathrm{Inn}(P)) \subseteq \pi_1(G)$, and $\pi_2(G) = \pi(n_i(P))$ for some $i$ such that $2 \leq i \leq s(P)$. Since $\pi_2(L) = \{73\}$, the number $n_i(P)$ equals to $73^k$ for some positive integer $k$. Moreover, by Lemma 4, we have $t(P) \geq t(L) - 1 = 5 - 1 = 4$ and $t(2, P) \geq 4$.

Prove that $P \cong E_6(2)$. Using Lemma 2 and Table 1, consider possibilities for $P$.

Let $P = A_2^{\epsilon}(q)$ for $\epsilon \in \{+, -\}$ and $q = 2^m > 4$ such that $(q - \epsilon 1)_3 = 3$. Since $n_2(P) = 73^k$, we have $(q^3 - \epsilon 1)/((q - \epsilon 1)(q - \epsilon 1, 3)) = (q^2 - \epsilon q + 1)/3 = 73^k$, i. e., $q^2 - \epsilon q + 1 = 3 \cdot 73^k$. Consider the remainders dividing by 8 of the left part and of the right part of the latter equality. The right part of the equality is compared with 3 modulo 8. Since $q = 2^m \geq 8$, the left part is compared with 1 modulo 8; a contradiction.

Let $P = {}^2D_n(q)$ for $n = 2^m \geq 4$ and even $q$. Since $n_2(P) = 73^k$, we have $q^n + 1 = 73^k$ and, therefore, $73^k - q^n = 1$. By Lemma 5, we have $k = 1$ and $q^n = 72$; a contradiction.

Let $P$ be one of the groups $E_8(q)$, $E_7(2)$, $E_6(2^m)$ or ${}^2E_6(2^m)$, where $2^m > 2$. This case does not appear since by Lemma 6, in this case $|\pi(P)| > |\pi(L)| = 8$.

Let $P$ be one of the sporadic simple groups, $A_2(4)$, or ${}^2E_6(2)$. This case does not appear since $73 \notin \pi(P)$.

Let $P = {}^2F_4(q)$, where $q = 2^{2m+1} > 2$. Then $2^{4m+2} - 2^{3m+2} + 2^{2m+1} - 2^{m+1} + 1 = 73^k$ or $2^{4m+2} + 2^{3m+2} + 2^{2m+1} + 2^{m+1} + 1 = 73^k$ for some natural number $k$. Note that the number $q - 1 = 2^{2m+1} - 1$ divides $|P|$, and the number $2^{2m+1} - 1$ has a primitive prime divisor which is contained in the set $\{3, 5, 7, 13, 17, 31\}$. Therefore, $2m + 1 \in \{3, 5\}$, i. e., $m$ is 1 or 2. But the numbers $2^6 - 2^5 + 2^3 - 2^2 + 1 = 37$, $2^6 + 2^5 + 2^3 + 2^2 + 1 = 109$, $2^{10} - 2^8 + 2^5 - 2^3 + 1 = 793$ and $2^{10} + 2^8 + 2^5 + 2^3 + 1 = 1321$ are not powers of 73; a contradiction.

$P = {}^2B_2(q)$, where $q = 2^{2m+1} > 2$. Since $t(P) = 4 = t(G) - 1$, by Lemma 4, each coclique of maximal size of $\Gamma(G)$ contains a coclique of maximal size of $\Gamma(P)$. But by [11], each coclique of maximal size of $\Gamma(P)$ contains 2 and any coclique of maximal size in $\Gamma(G)$ consists of odd primes; a contradiction.

Thus, we have proved that $P \cong E_6(2)$ and therefore, by [1], $G/F(G)$ is isomorphic to either $E_6(2)$ or $\mathrm{Aut}(E_6(2)) \cong E_6(2).2$. Suppose that $G/F(G) \cong \mathrm{Aut}(E_6(2))$. Then the centralizer $C$ in $P$ of an involutive graph automorphism of $P$ is isomorphic to $F_4(2)$ (see [3, 4.9.2]), and $|C|$ is divisible by 13 (see [1]), therefore, the vertices 2 and 13 are adjacent in $\Gamma(\mathrm{Aut}(E_6(2)))$. But the vertices 2 and 13 are not adjacent in $\Gamma(L)$; a contradiction. Thus, $G/F(G) \cong E_6(2)$.

Prove that $F(G) = 1$. Suppose that $O_p(G) \neq 1$ for some prime $p$. By Lemma 1, we have $p \in \pi_1(L)$. Since the Gruenberg–Kegel graphs of the groups $G$ and $G/(O_{p'}(G) \times \Phi(O_p(G)))$ are the same, without loss of generality we can assume that $F(G) = O_p(G)$ and $O_p(G)$ is an elementary abelian $p$-group.

Suppose that $p \in \{7, 13, 17\}$. Note that the vertices 5 and 31 are adjacent in $\Gamma(L)$ only with the vertices 2 and 3. By [1], we have $A_4(2) \cong PSL_5(2) < E_6(2)$, and the group $PSL_5(2)$ has a subgroup which is isomorphic to a Frobenius group of the shape $31 : 5$. Since $C_G(O_p(G)) = O_p(G)$, by Lemma 7, vertices $p$ and 5 are adjacent in $\Gamma(G)$; a contradiction.

Thus, $p \in \{2, 3, 5, 31\}$. Suppose that $p$ is odd. By [1], we have ${}^3D_4(2) < F_4(2) < E_6(2)$. Now by Lemma 8, an element of order $2^4 - 2^2 + 1 = 13$ from $G$ centralizes an element of order $p$ from $O_p(G)$, i. e., vertices 13 and $p$ are adjacent in $\Gamma(G)$ a contradiction.

Therefore, $p = 2$. By [1], we have $PSL_4(3) \cong A_3(3) < F_4(2) < E_6(2)$. Therefore a subgroup from $E_6(2)$ which is isomorphic to $PSL_4(3)$ acts on $O_2(G)$ with the property that an element of order 13 acts fixed-point-free on the elementary abelian 2-group $O_2(G)$. By Lemma 9, this contradicts to the table of 2-modular Brauer characters of the group $PSL_4(3)$ (see [5]). Thus, $F(G) = 1$ and, therefore, $G \cong L$.

The theorem is proved.

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